$d: \quad 2x+3y-6=0$

$F(3;3)$

$P(x;y) \,/\, \boxed{d(P,d)=\overline{PF}}$

$\overline{PF}=\sqrt{(x-3)^2+(y-3)^2}=\sqrt{x^2+9-6x+y^2+9-6y}=\sqrt{x^2+y^2-6x-6y+18}$

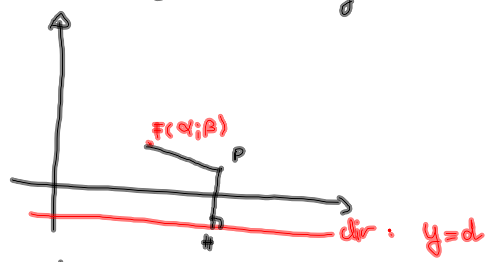
$d(P,d)=\dfrac{|2x+3y-6|}{\sqrt{13}}$

$\left(\sqrt{x^2+y^2-6x-6y+18}\right)^2=\left(\dfrac{|2x+3y-6|}{\sqrt{13}}\right)^2$

$x^2+y^2-6x-6y+18=\dfrac{4x^2+9y^2+36+12xy-24x-36y}{13}$

$13x^2+13y^2-78x-78y+234 \quad -4x^2-9y^2-36-12xy+24x+36y=0$

$9x^2+4y^2-12xy-54x-42y+198=0$



$P(x;y) \,/\, d(P,dir)=\overline{PF}$

$d(P,dir)=|y-d|$

$\overline{PF}=\sqrt{(x-\alpha)^2+(y-\beta)^2}$

$(|y-d|)^2=\left(\sqrt{(x-\alpha)^2+(y-\beta)^2}\right)^2$

$\cancel{y^2}+d^2-2dy=x^2-2\alpha x+\alpha^2+\cancel{y^2}+\beta^2-2\beta y$

mancano i termini: xy, y^2

$2\beta y-2dy=x^2-2\alpha x+\alpha^2+\beta^2-d^2$

$2(\beta-d)y=x^2-2\alpha x+\alpha^2+\beta^2-d^2$

$\beta-d\neq 0 \qquad y=\underbrace{\dfrac{1}{2(\beta-d)}}_{a}x^2-\underbrace{\dfrac{\cancel{2}\alpha}{\cancel{2}(\beta-d)}}_{b}x+\underbrace{\dfrac{\alpha^2+\beta^2-d^2}{2(\beta-d)}}_{c}$

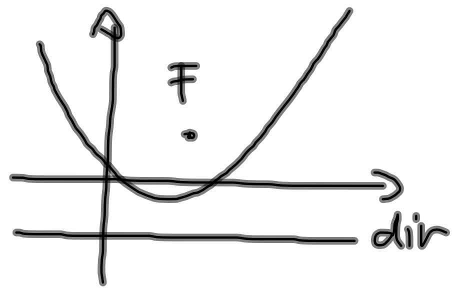
$\boxed{y=ax^2+bx+c}$

eq. parabola con asse di simmetria parallelo all'asse y in forma canonica

- se $\beta=d$ la parabola degenera in una retta
- $a\neq 0 \quad \forall\, F(\alpha,\beta)$ e $\forall$ dir

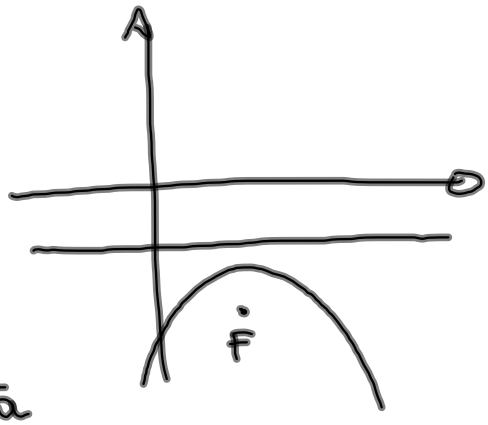
$$a = \frac{1}{2(\beta - d)}$$

$\beta - d > 0 \Rightarrow a > 0$



concavità secondo verso positivo asse y

$\beta - d < 0 \Rightarrow a < 0$

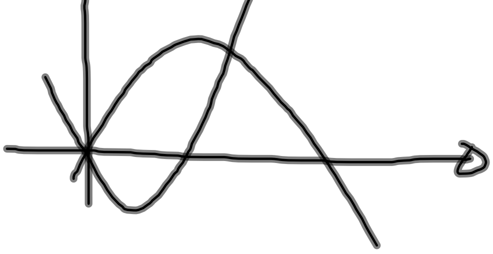


concavità rivolta secondo il verso negativo dell'asse y

- se $\alpha = 0$ ($F \in$ asse y) $\Rightarrow b = 0$

$$y = ax^2 + c$$

- se $c = 0$ $\quad y = ax^2 + bx$ la parabola passa x O



$y = ax^2 + bx + c$

dir : $y = d$

$F(\alpha, \beta)$